

Factoring RSA keys from certified smart cards: Coppersmith in the wild

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Nicko van Someren

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Problems with non-randomness

- ▶ 2012 Heninger–Durumeric–Wustrow–Halderman,
- ▶ 2012 Lenstra–Hughes–Augier–Bos–Kleijnung–Wachter.
- ▶ Factored tens of thousands of public keys on the Internet
... typically keys for your home router, not for your bank.
- ▶ Why? **Many deployed devices shared prime factors.**
- ▶ Most common problem: horrifyingly bad interactions between OpenSSL key generation, `/dev/urandom` seeding, entropy sources.
- ▶ The Heninger team has lots of material online at <http://factorable.net>

Nice followup student projects in data mining

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2. Check for common factors.
3. Write report that you've done the work and there are none.

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This started as such a student project on a very nice system:
MOICA: Certificate Authority of MOI (Ministry of the Interior).
In Taiwan all citizens can get a smartcard with signing and encryption ability to

- ▶ file personal income taxes,
- ▶ update car registration,
- ▶ make transactions with government agencies (property registries, national labor insurance, public safety, and immigration),
- ▶ file grant applications,
- ▶ interact with companies (e.g. Chunghwa Telecom).

Taiwan Citizen Digital Certificate

- ▶ Smart cards are issued by the government.
- ▶ FIPS-140 and Common Criteria Level 4+ certified.
- ▶ RSA keys are generated on card.
- ▶ About 3,002,000 certificates (all using RSA keys) stored on national LDAP directory. This is publicly accessible to enable citizen-to-citizen and citizen-to-commerce interactions.



Certificate of Chen-Mou Cheng

Data: Version: 3 (0x2)

Serial Number: d7:15:33:8e:79:a7:02:11:7d:4f:25:b5:47:e8:ad:38

Signature Algorithm: sha1WithRSAEncryption

Issuer: C=TW, O=XXX

Validity

Not Before: Feb 24 03:20:49 2012 GMT

Not After : Feb 24 03:20:49 2017 GMT

Subject: C=TW, CN=YYY serialNumber=0000000112831644

Subject Public Key Info:

Public Key Algorithm: rsaEncryption

Public-Key: (2048 bit) Modulus:

00:bf:e7:7c:28:1d:c8:78:a7:13:1f:cd:2b:f7:63:
2c:89:0a:74:ab:62:c9:1d:7c:62:eb:e8:fc:51:89:
b3:45:0e:a4:fa:b6:06:de:b3:24:c0:da:43:44:16:
e5:21:cd:20:f0:58:34:2a:12:f9:89:62:75:e0:55:
8c:6f:2b:0f:44:c2:06:6c:4c:93:cc:6f:98:e4:4e:
3a:79:d9:91:87:45:cd:85:8c:33:7f:51:83:39:a6:
9a:60:98:e5:4a:85:c1:d1:27:bb:1e:b2:b4:e3:86:
a3:21:cc:4c:36:08:96:90:cb:f4:7e:01:12:16:25:
90:f2:4d:e4:11:7d:13:17:44:cb:3e:49:4a:f8:a9:
a0:72:fc:4a:58:0b:66:a0:27:e0:84:eb:3e:f3:5d:
5f:b4:86:1e:d2:42:a3:0e:96:7c:75:43:6a:34:3d:
6b:96:4d:ca:f0:de:f2:bf:5c:ac:f6:41:f5:e5:bc:
fc:95:ee:b1:f9:c1:a8:6c:82:3a:dd:60:ba:24:a1:
eb:32:54:f7:20:51:e7:c0:95:c2:ed:56:c8:03:31:
96:c1:b6:6f:b7:4e:c4:18:8f:50:6a:86:1b:a5:99:
d9:3f:ad:41:00:d4:2b:e4:e7:39:08:55:7a:ff:08:
30:9e:df:9d:65:e5:0d:13:5c:8d:a6:f8:82:0c:61:
c8:6b

Exponent: 65537 (0x10001)

This project took a slightly different turn

HITCON 2012 (July 20–21):

Prof. Li-Ping Chou presents “Cryptanalysis in real life”
(based on work with Yun-An Chang and Chen-Mou Cheng)

Factored 103 Taiwan Citizen Digital Certificates
(out of 2.26 million keys with 1024 bits).

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End of story.

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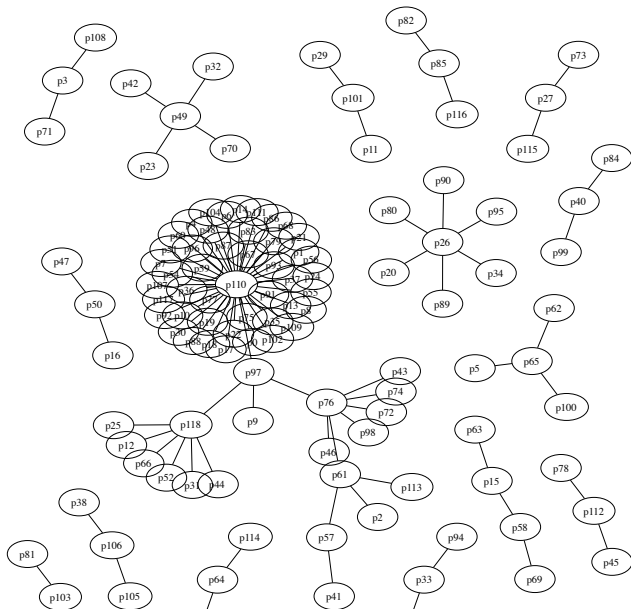
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End of story?

January 2013: Closer look at the 119 primes



How is this pattern generated?

1100100100100100001001001001001000100100100100101001001001001001
1001001001001001010010010010010001001001001001000010010010010010
0010010010010010100100100100100110010010010010010100100100100100
0100100100100100001001001001001000100100100100101001001001001001
1001001001001001010010010010010001001001001001000010010010010010
0010010010010010100100100100100110010010010010010100100100100100
0100100100100100001001001001001000100100100100101001001001001001
1001001001001001010010010010010001001001001001000010010011100101

How is this pattern generated?

Swap every 16 bits in a 32 bit word

```
0010010010010010 1100100100100100 1001001001001001 0010010010010010  
0100100100100100 1001001001001001 0010010010010010 0100100100100100  
1001001001001001 0010010010010010 0100100100100100 1001001001001001  
0010010010010010 0100100100100100 1001001001001001 0010010010010010  
0100100100100100 1001001001001001 0010010010010010 0100100100100100  
1001001001001001 0010010010010010 0100100100100100 1001001001001001  
0010010010010010 0100100100100100 1001001001001001 0010010010010010  
0100100100100100 1001001001001001 0010010011100101 0100100100100100
```


Prime generation

1. Choose a bit pattern of length 1, 3, 5, or 7 bits, repeat it to cover more than 512 bits, and truncate to exactly 512 bits.
2. For every 32-bit word, swap the lower and upper 16 bits.
3. Fix the most significant two bits to 11.
4. Find the next prime greater than or equal to this number.

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Factoring by trial division

Do this for any pattern:

0,1,001,010,011,100,101,110

00001,00010,00011,00100,00101,0011,00111,01000,01001,01010,...

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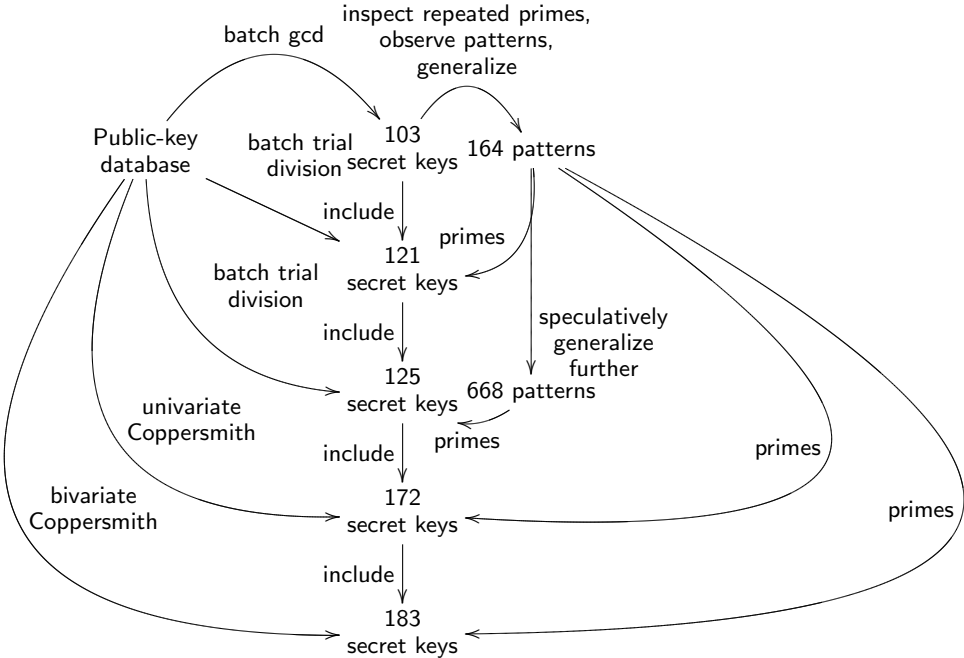
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Factored 4 more keys using patterns of length 9.



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Hypothesized failure:

- ▶ Hardware ring oscillator gets stuck in some conditions or does not output quickly enough.
- ▶ Card software not post-processing RNG output.

Important Lesson:

- ▶ Nontrivial GCD is not the only way RSA can fail with bad RNG.

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- ▶ Breaking RSA-1024 using Intel's new RDRAND_NSAKEY instruction.