High-Performance Modular Multiplication on the Cell Broadband Engine

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Outline

- Motivation and previous work
- Applications for multi-stream modular multiplication
- Background
    - Fast reduction with special primes
    - The Cell broadband engine
- Modular multiplications on the Cell
- Performance results
- Conclusions

Motivation

Modular multiplication is a performance critical operation in many cryptographic applications

- RSA
- ElGamal
- Elliptic curve cryptosystems

as well as in cryptanalytic applications

- computing elliptic curve discrete logarithms (Pollard rho)
- factoring integers (elliptic curve factorization method)

Measure the performance on the Cell.

Previous works

Misc. Platforms

Lots of performance results for many platforms

- GNU Multiple Precision (GMP) Arithmetic Library: almost all platforms (multiplicaton + reduction seperately),
- Bernstein et. al (Eurocrypt 2009): NVIDIA GPUs,
- Brown et al. (CT-RSA 2001): NIST primes on x_86.

On the Cell Broadband Engine

- The Multi-Precision Math (MPM) Library by IBM,

Optimize for one specific bit-size

- Costigan and Schwabe (Africacrypt 2009): special 255-bit prime,
- Bernstein et. al (SHARCS 2009): 195-bit generic moduli

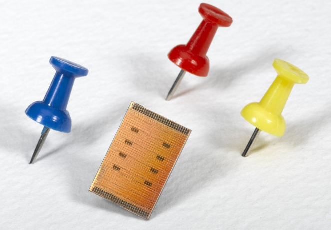
Contributions

What did I do?

Present high-performance multi-stream algorithms

- Montgomery multiplication,
- schoolbook multiplication,
- various special reduction schemes.

Implementation details (in C) are presented for a cryptologic interesting range 192 – 521 bits targeted at the Cell Broadband Engine.



Multi-Stream Modular Multiplication Applications

Modular exponentiations using a square-and-multiply algorithm.

Cryptography

- Exponentiations with the same random exponent:
  - ElGamal encryption (ElGamal, Crypto 1984),
  - Double base ElGamal - Damgård ElGamal (Damgård, Crypto 1991),
  - “Double” hybrid Damgård ElGamal (Kiltz et. al, Eurocrypt 2009).
- Batch decryption in elliptic curve cryptosystems

Cryptanalysis

- Pollard rho (elliptic curve discrete logarithm problem)
- Integer factorization (elliptic curve factorization method)

Special Primes

Faster reduction exploiting the structure of the special prime.

By US National Institute of Standards

Five recommended primes in the FIPS 186-3 (Digital Signature Standard)

$$\begin{aligned} P_{192} &= 2^{192} - 2^{64} - 1 \\ P_{224} &= 2^{224} - 2^{96} + 1 \\ P_{256} &= 2^{256} - 2^{224} + 2^{192} + 2^{96} - 1 \\ P_{384} &= 2^{384} - 2^{128} - 2^{96} + 2^{32} - 1 \\ P_{521} &= 2^{521} - 1 \end{aligned}$$

Prime used in Curve25519

Proposed by Bernstein at PKC 2006

$$P_{255} = 2^{255} - 19$$

Example: $P_{192} = 2^{192} - 2^{64} - 1$

$$0 \leq x < P_{192}^2, \quad 0 \leq x_H, x_L < 2^{192}, \quad x = x_H \cdot 2^{192} + x_L$$

$$x \equiv x_L + x_H \cdot 2^{64} + x_H \bmod P_{192}$$

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$$x_H \cdot 2^{64} < 2^{256}$$

$$x_H \cdot 2^{64} \equiv x_H \cdot 2^{64} \bmod 2^{192} + \left\lfloor \frac{x_H \cdot 2^{64}}{2^{192}} \right\rfloor \cdot 2^{64} + \left\lfloor \frac{x_H \cdot 2^{64}}{2^{192}} \right\rfloor \bmod P_{192}$$

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$s_1 = (c_5, c_4, c_3, c_2, c_1, c_0)$, $s_2 = (c_{11}, c_{10}, c_9, c_8, c_7, c_6)$,
$s_3 = (c_9, c_8, c_7, c_6, 0, 0)$, $s_4 = (0, 0, c_{11}, c_{10}, 0, 0)$,
$s_5 = (0, 0, 0, 0, c_{11}, c_{10})$ Return $s_1 + s_2 + s_3 + s_4 + s_5$

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$$\begin{aligned} s_1 &= (c_5, c_4, c_3, c_2, c_1, c_0), & s_2 &= (0, 0, c_7, c_6, c_7, c_6), \\ s_3 &= (c_9, c_8, c_9, c_8, 0, 0), & s_4 &= (c_{11}, c_{10}, c_{11}, c_{10}, c_{11}, c_{10}) \\ & & &\text{Return } s_1 + s_2 + s_3 + s_4 \end{aligned}$$

Solinas, technical report 1999

Note: this reduces to $[0, 4 \cdot P_{192}]$

Multiplication algorithms

Generic Moduli

Montgomery multiplication (with final subtraction)

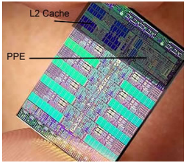
Special Moduli

Multiplication + special reduction

Size of the modulus:	192 - 521 bit
Multiplication method:	schoolbook

Investigate other methods (such as Karatsuba) is left as future work.

The Cell Broadband Engine



Cell architecture in the PlayStation 3 (@ 3.2 GHz):

- Broadly available (24.6 million)
- Relatively cheap (US$ 300)

The Cell contains

- eight “Synergistic Processing Elements” (SPEs)
  six available to the user in the PS3
- one “Power Processor Element” (PPE)
- the Element Interconnect Bus (EIB)
  a specialized high-bandwidth circular data bus

Cell architecture, the SPEs

The SPEs contain

- a Synergistic Processing Unit (SPU)
    - Access to 128 registers of 128-bit
    - SIMD operations
    - Dual pipeline (odd and even)
    - Rich instruction set
    - In-order processor
- 256 KB of fast local memory (Local Store)
- Memory Flow Controller (MFC)

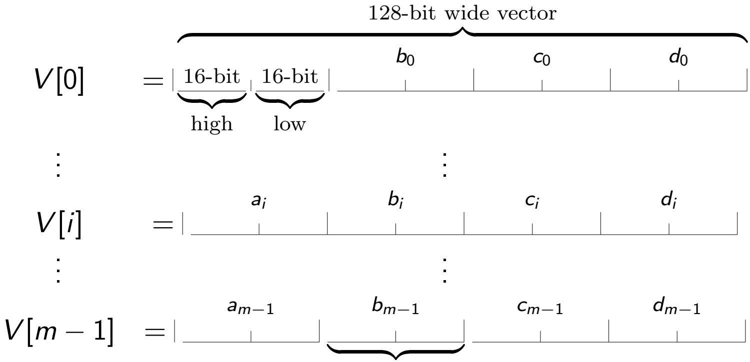
Programming Challenges

- Memory
    - The executable **and** all data should fit in the LS
    - *Or* perform manual DMA requests to the main memory (max. 214 MB)
- Branching
    - No “smart” dynamic branch prediction
    - Instead “prepare-to-branch” instructions to redirect instruction prefetch to branch targets
- Instruction set limitations
    - $16 \times 16 \rightarrow 32$ bit multipliers (4-SIMD)
- Dual pipeline
    - One odd and one even instruction can be dispatched per clock cycle.

Modular Multiplication on the Cell I

Four $(16 \cdot m)$-bit integers A, B, C, D represented in m vectors.

$$X = \sum_{i=0}^{m-1} x_i \cdot 2^{16 \cdot i}$$



128-bit wide vector

$V[0] = $ $\underbrace{\text{16-bit}}_{\text{high}}$ $\underbrace{\text{16-bit}}_{\text{low}}$ b_0 c_0 d_0

$\vdots$

$V[i] = $ a_i b_i c_i d_i

$\vdots$

$V[m-1] = $ a_{m-1} b_{m-1} c_{m-1} d_{m-1}

the most significant position of X_1 is located in

either the lower or higher 16-bit of the 32-bit word

Modular Multiplication on the Cell II

Implementation

- use the multiply-and-add instruction,
    - if $0 \leq a, b, c, d < 2^{16}$, then $a \cdot b + c + d < 2^{32}$.
- try to fill both the odd and even pipelines,
- are branch-free.

- Do not fully reduce modulo (m-bits) P,
- Montgomery and special reduction $[0, 2^m\rangle$,
- These numbers can be used as input again,
- Reduce to $[0, P\rangle$ at the cost of a single comparison $+$ subtraction.

Modular Multiplication on the Cell III

Special reduction $\rightarrow [0, t \cdot P\rangle \quad (t \in \mathbb{Z}$ and small$)$

How to reduce to $[0, 2^m\rangle$?

- Apply special reduction again
- Repeated subtraction (t times)

For a constant modulus m-bit P

Select the four values to subtract simultaneously
using `select` and `cmpgt` instructions and a look-up table.

Modular Multiplication on the Cell IV

For the special primes this can be done even faster.

t	$t \cdot P_{224} = t \cdot \left(2^{224} - 2^{96} + 1\right) = \{c_7, \ldots, c_0\}$						
	c_7	c_6	c_5	c_4	c_3	c_2	c_1	c_0
0	0	0	0	0	0	0	0	0
1	0	$2^{32} - 1$	$2^{32} - 1$	$2^{32} - 1$	$2^{32} - 1$	0	0	1
2	1	$2^{32} - 1$	$2^{32} - 1$	$2^{32} - 1$	$2^{32} - 2$	0	0	2
3	2	$2^{32} - 1$	$2^{32} - 1$	$2^{32} - 1$	$2^{32} - 3$	0	0	3
4	3	$2^{32} - 1$	$2^{32} - 1$	$2^{32} - 1$	$2^{32} - 4$	0	0	4

- $c_0 = t$, $c_1 = c_2 = 0$ and $c_3 = (\text{unsigned int})\ (0 - t)$.
- If $t > 0$ then $c_4 = c_5 = c_6 = 2^{32} - 1$ else $c_4 = c_5 = c_6 = 0$.
- Use a single `select`.

Modular Multiplication on the Cell V

$$P_{255} = 2^{255} - 19$$

Original approach

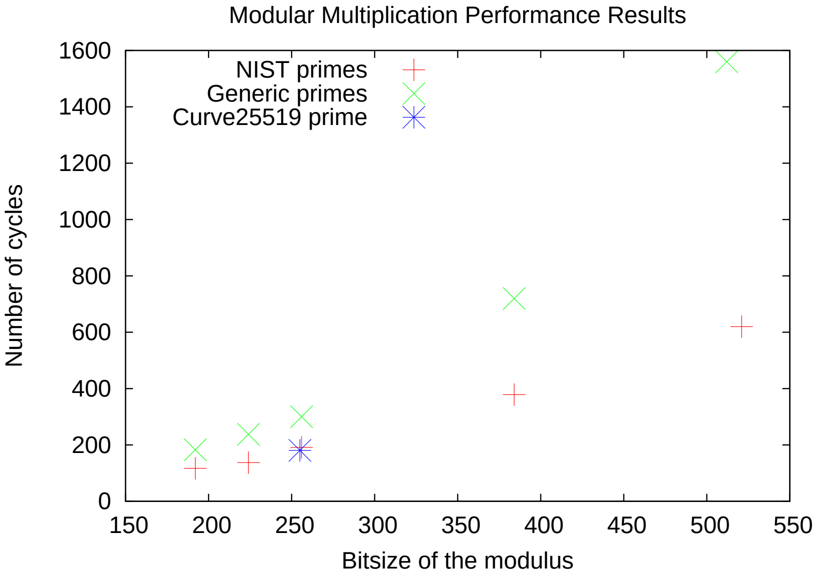
Proposed by Bernstein and implemented on the SPE by Costigan and Schwabe (Africacrypt 2009):

Here $x \in \mathbb{F}_{2^{255}-19}$ is represented as $x = \sum_{i=0}^{19} x_i 2^{\lceil 12.75i \rceil}$.

Redundant representation

- Following ideas from Bos, Kaihara and Montgomery (SHARCS 2009),
- Calculate modulo $2 \cdot P_{255} = 2^{256} - 38 = \sum_{i=0}^{15} x_i 2^{16}$,
- Reduce to $[0, 2^{256}\rangle$.

Performance Results



$$\frac{\text{Montgomery multiplication}}{\text{multiplication + fast reduction}} \approx 1.4 - 2.5$$

Comparison Special Moduli

Number of cycles for what?

- Measurements over millions of multi-stream modular multiplications,
- Cycles for a single modular multiplication,
- include benchmark overhead, function call, loading (storing) the input (output), converting from radix-2^{32} to radix-2^{16}.

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Special prime P_{255}

- Costigan and Schwabe (Africacrypt 2009), 255 bit.
- single-stream: 444 cycles (144 mul, 244 reduction, 56 overhead).
- multi-stream: 168 cycles.
  - no function call, loading and storing, "perfectly" scheduled (filled both pipelines)
- this work: 180 cycles ($< 168 + 56$),
- both approaches are comparable in terms of speed (on the Cell).

Comparison Generic Moduli

Generic 195-bit moduli

- Bernstein et al. (SHARCS 2009), multi-stream, 189 cycles,
- This work: multi-stream, 159 cycles for 192-bit generic moduli,
- Scaling: $(\frac{195}{192})^2 \cdot 159 = 164$ cycles.

Generic moduli

Bitsize	#cycles		
	New	MPM	uMPM
192	159	1,188	877
224	237	1,188	877
256	300	1,188	877
384	719	2,092	1,610
512	1,560	3,275	2,700

Conclusions

- We presented SIMD algorithms for Montgomery and schoolbook multiplication and fast reduction.
- Implementations are optimized for the Cell architecture.
- Implementation results for moduli of size 192 to 521 bits show that special primes are 1.4 to 2.5 times faster compared to generic primes.

Future work

- Try Karatsuba multiplication
- Further optimize Montgomery multiplication (almost finished)