Hash-based signatures II
Stateful and stateless signatures

Daniel J. Bernstein\textsuperscript{12} and Tanja Lange\textsuperscript{3}

\textsuperscript{1}University of Illinois at Chicago
\textsuperscript{2}Ruhr University Bochum
\textsuperscript{3}Eindhoven University of Technology

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Merkle’s (e.g.) 8-time signature system

Hash 8 one-time public keys into a single Merkle public key $P_{15}$.

\[ P_{15} = H(P_{13}, P_{14}) \]

\[ P_{13} = H(P_9, P_{10}) \]
\[ P_9 = H(P_1, P_2) \]
\[ P_1 \]
\[ S_1 \]
\[ P_2 \]
\[ S_2 \]
\[ P_3 \]
\[ S_3 \]
\[ P_4 \]
\[ S_4 \]
\[ P_{10} = H(P_3, P_4) \]
\[ P_5 \]
\[ S_5 \]
\[ P_6 \]
\[ S_6 \]
\[ P_11 = H(P_5, P_6) \]
\[ P_7 \]
\[ S_7 \]
\[ P_8 \]
\[ S_8 \]
\[ P_{12} = H(P_7, P_8) \]

\[ S_i \rightarrow P_i \] can be Lamport or Winternitz one-time signature system. Each such pair $(S_i, P_i)$ may be used only once.
Signature in 8-time Merkle hash tree

Signature of first message: \((\text{sign}(m, S_1), P_1, P_2, P_{10}, P_{14})\).

\[
\begin{align*}
P_{15} &= H(P_{13}, P_{14}) \\
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\end{align*}
\]
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Signature of first message: \((\text{sign}(m, S_1), P_1, P_2, P_{10}, P_{14})\).

Verify signature \(\text{sign}(m, S_1)\) with public key \(P_1\) (provided in signature). Link \(P_1\) against public key \(P_{15}\) by computing \(P'_9 = H(P_1, P_2), P'_{13} = H(P'_9, P_{10})\), and comparing \(H(P'_{13}, P_{14})\) with \(P_{15}\). Reject if \(H(P'_{13}, P_{14}) \neq P_{15}\) or if the signature verification failed.
Signature in 8-time Merkle hash tree

Signature of sixth message:

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Signature in 8-time Merkle hash tree

Signature of sixth message: $(\text{sign}(m', S_6), P_6, P_5, P_{12}, P_{13})$. 

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$P_{13} = H(P_9, P_{10})$

$P_9 = H(P_1, P_2)$

$P_1$

$S_1$

$P_2$

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$P_{10} = H(P_3, P_4)$

$P_3$

$S_3$

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$P_7$

$S_7$

$P_8$

$S_8$
Improvements to Merkle’s scheme

- Each public key (root of the tree) is good only for a fixed number of messages, typically $2^n$.
- The public key is very short: just one hash output. But each signature contains $n$ public keys along with the one-time signature.
- Computing the public key requires computing and storing $2^n$ one-time signature keys.
Trees of Merkle trees

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Trees of Merkle trees

$T_i$ are one-time signature keys.

↑ indicates input to hash function.
Trees of Merkle trees

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5 \quad T_6 \quad T_7 \quad T_8$

$PK \quad PK_5$

$T_5,1 \quad T_5,2 \quad T_5,3 \quad T_5,4 \quad T_5,5 \quad T_5,6 \quad T_5,7 \quad T_5,8$

$m \quad T_i$ and $T_{i,j}$ are one-time signature keys.

$\downarrow$ indicates signing.
Trees of Merkle trees

$T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow T_4 \rightarrow T_5 \rightarrow T_6 \rightarrow T_7 \rightarrow T_8 \rightarrow PK$

$m$ \(\downarrow\) $T_i$ and $T_{i,j}$ are one-time signature keys.

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No need to know $PK_5$ when generating the top tree.
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- Building trees of trees increases the signature length (one extra one-time signature per tree) and signing time. See PhD thesis of Andreas Hülsing for an optimized schedule of what to store and when to precompute. Only the top tree is needed to generate the public key.
Stateful hash-based signatures

- Only one prerequisite: a good hash function, e.g. SHA3-512. Hash functions map long strings to fixed-length strings. Signature schemes use hash functions in handling plaintext.
- Old idea: 1979 Lamport one-time signatures.
- 1979 Merkle extends to more signatures.

Pros:
- Post quantum
- Only need secure hash function
- Security well understood
- Fast

Cons:
- Biggish signature though some tradeoffs possible
- Stateful, i.e., ever reusing a subkey breaks security. Adam Langley “for most environments it’s a huge foot-cannon.”
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- Fast
- We can count: OS update, code signing, ... naturally keep state.

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Standardization progress

- CFRG has published 2 RFCs: **RFC 8391** and **RFC 8554**
  - **XMSS: eXtended Merkle Signature Scheme**
  - **Leighton-Micali Hash-Based Signatures**

- NIST has standardized XMSS and LMS.
- Only concern is about statefulness in general.
- ISO SC27 JTC1 WG2 has started a study period on stateful hash-based signatures.
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- These signatures are stateful, need to remember which leaf signature was used.

By the birthday paradox we need $2^{256}$ leaves! Cannot precompute this tree...
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- Cannot precompute this tree . . .
Huge trees (1987 Goldreich), keys on demand (Levin)

Signer chooses random \( r \in \{2^{255}, 2^{255} + 1, \ldots, 2^{256} - 1\} \), uses one-time public key \( T_r \) to sign message; uses one-time public key \( T_i \) to sign \( (T_{2i}, T_{2i+1}) \) for \( i < 2^{255} \).

Generates \( i \)th secret key deterministically as \( H_k(i) \) where \( k \) is master secret. Important for efficiency.
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![Diagram of tree structure](image)

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Use Goldreich to create stateless hash-based signatures

0.6 MB for hash-based Goldreich signature using short-public-key Winternitz-16 one-time signatures.

Would dominate traffic in typical applications, and add user-visible latency on typical network connections.
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Example:
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Example:
HTTPS typically sends multiple signatures per page.
1.8 MB average web page in Alexa Top 1000000.
Can we do with fewer leaves?

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Change definition of $H$ to have many components

$$H(m) = (h_0, h_1, \ldots, h_{k-1}),$$

where each $h_i \in \{0, 1, 2, \ldots, t - 1\}$ for some $t$.

Collisions mean that all $h_i$ match.
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**r-subset resilience**

Let $H(m_j) = (h_{j,0}, h_{j,1}, \ldots, h_{j,k-1})$.

$H$ is $r$-subset-resilient if given $H(m_1), H(m_2), \ldots, H(m_r)$ the probability of finding $m'$ with $H(m') = (h'_0, h'_1, \ldots, h'_{k-1})$ and $h_f \in \{h_{j,i} | 0 \leq i < k, 1 \leq j \leq r\}$ for $0 \leq f < k$ is negligible.
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The same leaf public key can be used for $r + 1$ signatures if $H$ if $r$-subset-resilient.
Few-times signature HORS
(Hash to Obtain Random Subset)

General parameters:
- Integer parameters $k$, $t$, $\ell$.
- Hash function $H : \{0, 1\}^* \rightarrow \{0, 1\}^{k \cdot \log_2 t}$.
- One-way function $f : \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell$.

KeyGen:
- Picks $t$ strings $s_i \in \{0, 1\}^\ell$, compute $v_i = f(s_i)$ for $0 \leq i < t$.
- Public key $P = (v_0, v_1, \ldots, v_{t-1})$; secret key $S = (s_0, s_1, \ldots, s_{t-1})$.

Sign $m \in \{0, 1\}^*$:
- Compute $H(m) = (h_0, h_1, \ldots, h_{k-1})$, where each $h_i \in \{0, 1, 2, \ldots, t-1\}$.
- Signature on $m$ is $\sigma = (s_{h_0}, s_{h_1}, s_{h_2}, \ldots, s_{h_{k-1}})$.

Verify:
- Compute $H(m) = (h_0, h_1, \ldots, h_{k-1})$ and $(f(s_{h_0}), f(s_{h_1}), f(s_{h_2}), \ldots, f(s_{h_{k-1}}))$.
- Verify that $f(s_{h_i}) = v_{h_i}$ for $0 \leq i < t$. 
HORS exercises, assume $H$ is surjective

1. Let $\ell = 80$, $t = 2^5$, and $k = 3$. How large (in bits) are the public and secret keys? How large is a signature? How many different signatures can the signer generate for a fixed key pair as $H(m)$ varies? Ignore that $s$-values could collide.

2. The same public key can be used for $r + 1$ signatures if $H$ is $r$-subset-resilient.
   Even for $r = 1$, i.e. after seeing just one typical signature, an attacker has an advantage at creating a fake signature. What are the options beyond exact collisions in $H$?

3. Let $\ell = 80$, $t = 2^5$, and $k = 3$. Let $m$ be a message so that $H(m) = (h_0, h_1, h_2)$ satisfies that $h_i \neq h_j$ for $i \neq j$. You get to specify messages that Alice signs. You may not ask Alice to sign $m$. State the smallest number of HORS signatures you need to request from Alice in order to construct a signature on $m$? How many calls to $H$ does this require on average? You should assume that $H$ and $f$ do not have additional weaknesses beyond having too small parameters. Explain how you could use under 1000 evaluations of $H$ if you are allowed to ask for two signatures.
Ingredients of SPHINCS (and SPHINCS-256)

Drastically reduce tree height (to 60).
Replace one-time leaves with few-time leaves.
Optimize few-time signature size plus key size.
New few-time HORST (HORS with trees), improving upon HORS.
Use hyper-trees (12 layers), as in GMSS.
Use masks, as in XMSS and XMSS\textsuperscript{MT}, for standard-model security proofs.
Optimize short-input (256-bit) hashing speed.
Use sponge hash (with ChaCha12 permutation).
Use fast stream cipher (again ChaCha12).
Vectorize hash software and cipher software.

See paper for details: sphincs.cr.yp.to

Updated version is NIST submission SPHINCS+ https://sphincs.org/.