Attention: one-line answers using a computer algebra system do not count. But it is a good moment to familiarize yourself with some system(s) so that you know how to solve similar problems for real life examples and to verify your answers. You may use a computer algebra system to compute subresults, such as factorizations, modular reduction, multiplication, squaring.

1. Show that \( M_3(\mathbb{Z}/2) \) forms a ring, where \( M_3(\mathbb{Z}/2) \) denotes the set of all \( 3 \times 3 \) matrices with entries in \( \mathbb{Z}/2 \). Is this ring commutative?

2. Compute \( \varphi(37800) \).

3. Compute \( \varphi(1939201349958859167498240) \).

4. Give all elements in \( (\mathbb{Z}/12)^\times \).

5. Give all elements in \( (\mathbb{Z}/21)^\times \).

6. Execute the RSA key generation where \( p = 239 \), \( q = 433 \), and \( e = 23441 \).

7. RSA-encrypt the message 23 to a user with public key \( (e, n) = (17, 11584115749) \). Document how you compute the exponentiation.