Ex. We compute $\gcd(2024, 748)$ using repeated division with remainder:

$$2024 = 748 \cdot 2 + 528$$
$$748 = 528 \cdot 1 + 220$$
$$528 = 220 \cdot 2 + 88$$
$$220 = 88 \cdot 2 + 44 \longleftarrow \gcd = 44$$
$$88 = 44 \cdot 2 + 0$$

Exercise: Compute the gcd of $\overset{a=}{1278}, \overset{b=}{234}$:

$$1278 = 234 \cdot 5 + 108$$
$$234 = 108 \cdot 2 + 18 \longleftarrow \gcd = 18.$$
$$108 = 18 \cdot 6 + 0$$

$$a = 5 \cdot b + 108 \iff 108 = a - 5b$$
$$b = 234 = (a - 5 \cdot b) \cdot 2 + 18$$

$$b = 2a - 10b + 18 \iff 18 = -2a + 11b$$

By ~~re-substituting.~~ substitution, we got a linear combination of a and b which is equal to the gcd of a and b. This leads to the Extended Euclidean Algorithm (EEA).

Theorem 2.4 Let a, b be positive integers. Then the equation
$$a \cdot u + b \cdot v = \gcd(a, b)$$
always has solutions in integers u and v.