

## Exercise sheet 2, 24 November 2016

Some convenient computer algebra systems are sage <http://sagemath.org/> and for small computations Pari-GP <http://pari.math.u-bordeaux.fr/>. I made a sage “cheat sheet” <http://hyperelliptic.org/tanja/teaching/alg14/sage-ref.pdf> for the algebra class.

If you know how to use Mathematica chances are that there are also some functions provided.

1. For the following LFSR descriptions find the characteristic polynomial, the order of the associated matrix and for all starting vectors the period length (only need this for one representative for each sequence); one sequence should be started at  $S_0 = (0, 0, \dots, 0, 1)$ . For each of them try to factor the characteristic polynomial over  $\mathbb{F}_2$ . Can you find any relation between the order of the sequence and the largest of the least periods?
  - (a)  $s_{k+2} = s_k + s_{k+1}$ ;
  - (b)  $s_{k+3} = s_k + s_{k+2}$ ;
  - (c)  $s_{k+3} = s_k + s_{k+1} + s_{k+2}$ ;
  - (d)  $s_{k+7} = s_{k+6} + s_{k+5} + s_{k+1} + s_k$ ;
  - (e)  $s_{k+10} = s_{k+7} + s_{k+2} + s_{k+1} + s_k$ .
2. The sequence  $s_{k+2} = s_k + s_{k+1}$  over the integers with starting values  $s_0 = 0, s_1 = 1$  is called the Fibonacci sequence. Compute the first 10 elements. Factor the characteristic polynomial of this sequence and call the roots  $\alpha$  and  $\bar{\alpha}$ . Compute  $\alpha^j + \bar{\alpha}^j$  for  $0 \leq j \leq 10$ . What do you notice?
3. Can you find a similar result for the sequences over  $\mathbb{F}_2$ ?